occurring about 12.5 or 13 GHz, but there is no indication of other than narrow-band performance.

Finally, two unsuccessful results should be mentioned briefly. Some results of empirical 4-port circulator design using full-height structures are available in the literature [6], [7], but attempts to predict these results were unsuccessful. This is attributed to a lack of reliable ferrite data, and the sensitivity of 4-port performance to small changes in material properties and dimensions. Also, an unsuccessful attempt was made to obtain a good (predicted) 4-port circulator using the design procedure suggested by Helszajn and Buffler [8]. Further investigations are under way.

IV. Discussion and Conclusions

Davies' theory of the symmetrical m-port nonreciprocal waveguide junction has been successfully extended to a higher approximation for $m=3$ and 4. The SA agrees very well and resolves some of the previously unexplained discrepancies displayed by the FA. The SA provides some of the "fine structure" and provides a more realistic estimate of bandwidth and loss between ports. This is particularly useful for the potentially broad-band thin-pin 3-port structures.

The penalty we have paid for improved calculations is an increase in computer time necessary to make them. The increase in the number of terms in n requires many more calculations and, especially for the more complex structures, the increase in computation time is marked. As examples let us consider a simple junction, the 3-port with a ferrite post, and then the more complicated 4-port with a ferrite post, metal pin, and dielectric sleeve. With the 3-port, the FA and SA take approximately 0.55 and 1.65 min, respectively, for a set of computations over the waveguide bandwidth. With the 4-port, the times are approximately 1.63 and 4.22 min, respectively. These times include approximately 0.180 min for the graphical printout in the form of a simulated swept-frequency plot. The programs might be made more efficient, and the machine was a Burroughs B-5500.

It should be emphasized that we now have a proven analysis technique for a particular class of waveguide structures; we do not have a circulator design program. Any successful design that is developed using this program is still likely to be the result of experience just as it would be in the laboratory, but the difference is that it would have been found in less time or it would be the best of a larger number of attempts. However, one would still not have any information as to whether this was the "best" design in any sense of the word.

Two avenues may now be followed. A performance figure of merit could be defined and optimization routines developed that would automatically maximize the figure of merit. This is probably the most straightforward extension, and one that will fill a real need. It will, however, be complicated by the many parameters present in all potentially attractive structures, the complexity of the behavior with parameter changes (particularly in the 4-port), and the fact that a global maximum is required.

The authors feel that while analysis and optimization are the only good design procedures within reach, the real requirement is for a successful circulator synthesis procedure.

References

[1] B. A. Auld, "The synthesis of symmetrical waveguide circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-7, pp. 238–246, Apr. 1959.
[2] J. B. Davies, "An analysis of the m-port symmetrical H-plane waveguide junction with central ferrite post," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-10, pp. 596–604, Nov. 1962.
[3] J. B. Castillo and L. E. Davis, "Computer-aided design of three-port waveguide junction circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 25–34, Jan. 1970.
[4] ——, "Identification of spurious modes in circulators," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-19, pp. 112–113, Jan. 1971.
[5] J. E. Pippin, "Ferrites—1968," *Microwave J.*, Apr. 1968; see fig. 5(a)–(c).
[6] L. E. Davis, M. D. Coleman, and J. J. Cotter, "Four-port-crossed-waveguide junction circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 43–47, Jan. 1964.
[7] G. S. Sidhu and O. P. Gandhi, "A four-port waveguide junction circulator and effects of dielectric loading on its performance," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-13, pp. 388–389, May 1965.
[8] J. H. Helszajn and C. R. Buffler, "Adjustments of the 4-port single junction circulator," *Radio Electron. Eng.*, vol. 35, no. 6, 1968.

Uniform Multimode Transmission Lines

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Abstract—**A consistent normal-mode theory for coupled uniform transmission lines is developed which properly accounts for the fact that some propagation constants may be equal. Explicit responses are calculated. The results are interpreted in terms of transmission-line networks which consist of simple lines and which are coupled via conductor sharing.**

The design and understanding of high-speed computer circuitry require the transient solution of the n-wire uniform transmission-line problem. The interconnections are normally treated as coupled lossless lines with a single propagation constant. If, however, one (or more) of the system members is loaded periodically, the model fails since several propagation constants may occur. Thus there is need for an analysis that allows for partial eigenvalue degeneracy and yields the transient response.

It is assumed that the n-wire and ground system is described by [1]

$$\left.\frac{dV}{dx}\right] = -[Z]I] \tag{1a}$$

$$\left.\frac{d}{dx} I\right] = -[Y]V] \tag{1b}$$

$$\left.\frac{d^2}{dx^2} V\right] = [Z][Y]V]. \tag{1c}$$

Here the conductor-to-ground voltages and the conductor currents are Laplace-transformed quantities. The matrices $[Z]$ and $[Y]$ are x independent. The propagation constants are the solutions of (2) [2]:

$$\left| [Z][Y] - \gamma^2[\delta_{ij}] \right| = 0. \tag{2}$$

To form the modal matrix, it is assumed initially that the n eigenvalues of $[\Gamma]=[Z][Y]$ are distinct. Then the solution for the kth mode for a system that extends from $x=0$ to ∞ becomes

$$V_k] = [\epsilon^{-\gamma_k x}]V_k(0)] \tag{3}$$

where the propagation matrix is diagonal. The total voltage at $x=0$, the sending end, becomes

$$\left.\sum_{k=1}^{n} V_k(0)\right] = [\alpha][V_{ii}(0)]1] \tag{4}$$

where $[V_{ii}(0)]$ is a diagonal matrix with elements $V_{kk}(0)$, i.e., the voltage from conductor k to ground for the kth mode. This form is possible if the modal matrix is constructed as follows. Substitution of γ_k into (1c) and suppression of the kth row of Γ yields

$$\begin{aligned} -\Gamma_{ik}]V_{kk} &= [A]V_{ik}], && i \neq k \\ A_{ii} &= \Gamma_{ii} - \gamma_k^2, && i \neq k \\ A_{ij} &= \Gamma_{ij}, && i \neq j,\ i \neq k,\ j \neq k. \end{aligned} \tag{5}$$

This equation may be solved for the eigenvector $V_k]$. In particular,

$$\alpha_{i,k} \equiv \frac{V_{ik}}{V_{kk}} \tag{6}$$

defines the members of the modal matrix $[\alpha]$. The voltages on the infinite system become

$$V] = [\alpha][V_{ii}][\epsilon^{-\gamma x}]1] \qquad [\epsilon^{-\gamma x}]_{ij} = \epsilon^{-\gamma_i x}\delta_{ij}. \tag{7}$$

The corresponding total current is readily shown to be

$$I] = [Y][\alpha][\gamma]^{-1}[\epsilon^{-\gamma x}][V_{ii}]1]. \tag{8}$$

The assumption that the system is excited at $x=0$ by $I_S]$ and ter-

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minated by $[Y_S]$ yields

$$[\alpha][V_{ii}(0)]1] = ([Y_S] + [Y][\alpha][\gamma]^{-1}[\alpha]^{-1})^{-1}I_S]. \quad (9a)$$

Here the apparent characteristic impedance is readily identified and agrees with Amemiya's result [3], [4]. The occurrence of the inverse modal matrix, however, would suggest difficulties with a system that is partially degenerate. This, of course, is the reason for the distinctness assumption. It is also the reason for the particular form of the modal matrix formation of (5). To clarify this, the fully degenerate case may be considered. If, for this case, $[V_{ii}]$ is interpreted as the voltage of line i to ground, i.e., if the word "modal" is ignored, and if $[\alpha]=[1]$, then (9a) becomes

$$V_{ii}(0)] = ([Y_S] + [Y][\gamma]^{-1})^{-1}I_S] \quad (9b)$$

which is exactly the fully degenerate solution. This would suggest that the modal matrix for the partially degenerate system may be constructed by partitioning. Thus if the first m eigenvalues are distinct and $s=n-m$ eigenvalues are equal, then $[\alpha]$ becomes

$$[\alpha] = \left[\begin{array}{c|c} \alpha(n,m) & \begin{array}{c} 0 \\ \hline 1(s,s) \end{array} \end{array}\right]. \quad (10)$$

Here the quantities in the parentheses denote the number of rows and columns of the submatrices. This form of $[\alpha]$ may be shown to be correct by rigorous calculations. It is convenient because it is nonsingular and because degeneracy may be introduced very simply by setting selected elements of α to zero. The earlier assumption of nondegeneracy is thus removed.

The reflection coefficients at the receiving end are due to the termination $[Y_R]$ at $x=L$. A straightforward evaluation yields

$$[V_{RR}]1] = [\epsilon^{-\gamma L}]([Y][\alpha][\gamma]^{-1} + [Y_R][\alpha])^{-1}([Y][\alpha][\gamma]^{-1} - [Y_R][\alpha])[\epsilon^{-\gamma L}][V_{ii}]1]. \quad (11)$$

The reflection coefficient at the receiving end is, therefore,

$$[\rho_R] = [\alpha]^{-1}[[Y_0] + [Y_R]]^{-1}[[Y_0] - [Y_R]][\alpha] \quad (12)$$

where

$$[Y_0] = [Y][\alpha][\gamma]^{-1}[\alpha]^{-1}.$$

The reflection coefficient at the sending end, $x=0$, may be obtained by subscript exchange. The unknown amplitudes become

$$\left.\frac{V_{kk}}{V_i}\right] = [\alpha][[\epsilon^{-\gamma x}] + [\epsilon^{-\gamma(L-x)}][\rho_R][\epsilon^{-\gamma L}]][[1] - [\rho_S][\epsilon^{-\gamma L}][\rho_R][\epsilon^{-\gamma L}]]^{-1}[\alpha]^{-1}([Y_S] + [Y_0])^{-1}I_S]. \quad (13)$$

Here the first m entries into the column vector denote the modal voltages in mode k on conductor k to ground. The remaining entries are the amplitudes from conductor $i=m+1, m+2, \cdots, n$ to ground which share the single propagation constant γ_{m+1}.

The physical interpretation of the analysis is of interest and worthy of comment.

Each mode has its own characteristic impedance $[Y][\gamma_k]^{-1}$. This mode characteristic impedance may be interpreted in terms of n simple transmission lines from the conductors to ground and $(n/2)(n-1)$ simple transmission lines that exist between conductors. Thus this modal network consists of separate transmission lines that are coupled by conductor sharing. Since there are m of these networks, a total of $(mn/2)(n+1)$ simple lines are involved in the nondegenerate part of the system. However, since one is talking about a normal mode, only one line voltage can be determined by the boundary conditions in each mode. The degenerate part of the transmission system consists of $(s/2)(s+1)$ simple lines. This system decouples itself from the main system by not exciting or grounding those members that do not share the degeneracy. However, since its eigenvalue is degenerate, s line voltages are determined by the boundary conditions. Thus for a single degeneracy the system reduces to $m+1$ subsystems and a total of $(n/2)(n+1)+[(n-m)/2](n-m+1)$ distinct transmission lines in which exactly n voltages are determined by the boundary conditions. The total system can be terminated by a passive network. This does not depend on the fact that the system is lossless. This reflectionless termination has the ability to transfer energy from one mode to the other to obtain a proper match. In the general case, the modal excitation is a time-dependent quantity governed by the forcing function, the boundary conditions, and the modal characteristic impedances and not by coupling between the modes. This statement is supported readily by expanding (13) in terms of multiple reflections that are useful for transient calculations and by noting that (9) is exactly a lumped-circuit calculation. The implication is and should be that the entire analysis may be based on calculations that occur at the boundary and in which transmission-line concepts are used only in the sense that cause and effect are time delayed and that currents and voltage are related via the characteristic impedance.

References

[1] S. Hayashi, *Surges on Transmission Lines*. Kyoto, Japan: Denki-Shoin, 1955.
[2] E. A. Guillemin, *The Mathematics of Circuit Analysis*. New York: Wiley, 1949.
[3] H. Amemiya, "Time-domain analysis of multiple parallel transmission lines," *RCA Rev.*, vol. 28, pp. 241–276, 1967.
[4] ——, "Matched termination network for multiple parallel transmission lines," *Electron. Lett.*, vol. 3, pp. 13–14, 1967.
[5] H. Hagiwara and S. Okugawa, "Time-domain analysis of multiconductor exponential lines," *Proc. IEEE* (Lett.), vol. 56, pp. 1111–1112, June 1968.

Computation of the Characteristics of Coplanar-Type Strip Lines by the Relaxation Method

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***Abstract*—The characteristics of new strip lines (i.e., a single strip conductor and a two symmetrical strip-conductor coplanar-type strip line, which consist of single- or two-center strip conductors and ground plates on a dielectric substrate and outer ground conductor) are calculated by the relaxation method. The effect of the outer ground conductor on these lines is analyzed, and the characteristic impedance and velocity ratio are determined. The characteristic impedance is determined experimentally, and the maximum values of the discrepancies compared with the calculated values of each of the lines are 2.0–3.0 percent.**

Introduction

Microwave circuits used in a communication satellite, for example, require light weight, small size, and high reliability, so the strip line is suited to these needs. The characteristic impedance and phase-velocity ratio of conventional triplate strip lines are determined by the thickness of the dielectric substrate and its relative dielectric constant, by the width of the strip conductors, and by the height of the line. In order to obtain a smaller line when using the same dielectric substrate and same height of line, or to obtain a more versatile line, different types of new lines must be considered.

The coplanar waveguide (CPW) is very attractive, and it is analyzed in open boundary by using conformal mapping [1]. But closed boundary lines are needed for high-gain amplifier circuits, and lines having side walls can help to miniaturize microwave circuits.

In this short paper, two new types of strip lines [i.e., the single strip-conductor coplanar-type strip line (S-CPS), which has a center strip conductor and ground plates on dielectric substrate as shown in Fig. 1(a), and the two symmetrical strip-conductor coplanar-type strip line (T-CPS), which is shown in Fig. (b)] are analyzed.

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